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One loop calculation of SUSY Ward-Takahashi identity on lattice with Wilson fermion

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Abstract

One loop correction to the SUSY Ward-Takahashi identity is calculated on lattice with Wilson fermion. The supersymmetry on lattice is broken explicitly by the gluino mass and the lattice artifact. We should fine tune parameters in the theory to the point given by the additive mass correction in order to eliminate the breaking effect of lattice artifact. It is shown that the additive mass correction appearing from the SUSY Ward-Takahashi identity coincide with that from the axial $U(1)_R$ symmetry as was suggested by Curci and Veneziano. Two important symmetries of the super Yang-Mills theory can be recovered simultaneously in the continuum with a single fine tuning.

Operator mixing of the supercurrent are also investigated. We find that the supercurrent mixes only with a gauge invariant current T_μ which is related to the gamma-trace anomaly.

I. INTRODUCTION

There has been a great progress in nonperturbative understanding of the low energy behavior of the $N = 1$ supersymmetric QCD [1]. The analysis is based on the global symmetry and the holomorphy of the superpotential and we can derive the nonperturbative form of superpotential. This is quite satisfactory when we investigate the vacuum structure of the theory. However when it is required to understand the low energy particle spectrum including excited states or an influence of the Kähler potential this method is insufficient and some other nonperturbative method is required.

The lattice regularization when applied to a supersymmetric theory breaks its supersymmetry explicitly. This is mainly caused by the artifact of the lattice regularization itself and the fermion problem on lattice. However ability of the lattice field theory to perform the path integral nonperturbatively with Monte-Carlo method is so fascinating that several efforts has been made to resolve the difficulty of SUSY on lattice. These attempts are classified into two types. One is to realize a SUSY on the lattice which corresponds to the ordinary supersymmetry in the continuum limit [2–4]. Although this method is beautiful in construction and can extract peculiar feature of the model due to supersymmetry before taking the continuum limit in principle , it is applicable only to the free Wess-Zumino model up to now. In order to treat the $N = 1$ supersymmetric Yang-Mills (SYM) theory, whose component fields (gluon and gluino) are forced to stay at different places (links and sites) on lattice to keep the gauge symmetry, we need the second method. In this method we do not persist in the supersymmetry and discretize the theory straightforwardly making use of the well known actions on lattice. The SUSY is recovered only in the continuum [5]. This restoration of supersymmetry is not automatic and the discussion in Ref. [5] is as follows.

The SYM theory has two important global symmetries in the continuum. One is supersymmetry and the other is axial $U(1)_R$ symmetry which is broken by the anomaly. Both the symmetries are broken explicitly when the theory is regularized on lattice with the Wilson plaquette action for gluon and the Wilson fermion action for gluino. The source of this symmetry breaking is classified into the introduction of gluino mass which cannot be forbidden in the Wilson fermion and the lattice artifacts of discretization. This explicit breaking effect of the lattice artifacts is given by irrelevant operators in the Ward-Takahashi (WT) identity and vanishes in the continuum at tree level. However when a quantum correction comes into play this term usually produces an additive corrections. It is required to fine tune several parameters of the theory in order to recover the symmetry in the continuum. Although this fine tuning should be performed independently for each symmetries, we have only one parameter (gluino mass) in $N = 1$ SYM which we can freely tune. It is discussed by Curci and Veneziano [5] that the both symmetries are restored simultaneously with a single fine tuning of the gluino mass to the chiral $U(1)_R$ symmetric point. Several Monte-Carlo studies of SYM theory have been done along this line [6–11] to reproduce the prediction of the low energy effective theory [12,13].

In this paper we formulate the $N = 1$ SYM theory on lattice with the Wilson plaquette and the Wilson fermion action according to Ref. [5]. We calculate the one loop correction to the Ward-Takahashi identity of both the SUSY and $U(1)_R$ symmetry perturbatively in the gauge variant Green function. It is shown that the additive mass correction appearing from the SUSY WT identity coincide with that from the axial $U(1)_R$ symmetry as was

suggested. This means that the both symmetries of the super Yang-Mills theory can be recovered simultaneously in the continuum with a single fine tuning of the gluino mass. We also investigate the mixing behavior of the supercurrent with the on-shell condition for gluino momentum and mass. The supercurrent mixes with the gauge invariant operator T_μ as was predicted in Ref. [5]. This current is related to the gamma-trace anomaly of the supercurrent. An extra mixing with gauge variant operators occurs too. This is because we used gauge variant Green function in our calculation. However these extra mixings vanish by setting the renormalized gluino mass to zero together with the on-shell condition.

This paper is organized as follows. In Sec. II we introduce the lattice SYM action and the Feynman rules relevant for the one loop calculation. In Sec. III we define the super transformation on lattice and give the concerning SUSY WT identity. The WT identity for $U(1)_R$ symmetry is also given in this section. Sec. IV and V are devoted to the calculation of quantum correction at one loop level for the axial and SUSY Ward-Takahashi identity. Our conclusion is summarized in Sec. VI.

The physical quantities are expressed in lattice units and the lattice spacing a is suppressed unless necessary. We take SU(N_c) gauge group with the gauge coupling g, the generator T^a and the structure constant f^{abc}. The normalization is given as $\mathrm{tr}(T^aT^b) = \frac{1}{2}\delta^{ab}$.

II. ACTION AND FEYNMAN RULE

The SYM theory is given as a minimally gauge coupled massless adjoint Majorana fermion system in the continuum. In this paper we adopt the following lattice regularization procedure [5]. The gauge part is given by a standard four dimensional Wilson plaquette action,

$$S_{\mathrm{gluon}} = \sum_n \sum_{\mu\nu} -\frac{1}{g^2} \mathrm{Re}\,\mathrm{tr}\left(U_{n,\mu}U_{n+\hat{\mu},\nu}U^\dagger_{n+\hat{\nu},\mu}U^\dagger_{n,\nu}\right). \tag{II.1}$$

The gluino part is given by the Wilson fermion,

$$\begin{aligned} S_{\mathrm{gluino}} &= \sum_n \mathrm{tr}\Big[\frac{1}{2}\overline{\psi}(n)\left(-r+\gamma_\mu\right)U_\mu(n)\psi(n+\mu)U^\dagger_\mu(n) \\ &\qquad + \frac{1}{2}\overline{\psi}(n+\hat{\mu})\left(-r-\gamma_\mu\right)U^\dagger_\mu(n)\psi(n)U_\mu(n) + (M+4r)\overline{\psi}(n)\psi(n)\Big] \\ &= \sum_n \mathrm{tr}\Big[\overline{\psi}(n)\left(-r+\gamma_\mu\right)U_\mu(n)\psi(n+\mu)U^\dagger_\mu(n) + (M+4r)\overline{\psi}(n)\psi(n)\Big], \end{aligned} \tag{II.2}$$

where gluino filed $\psi = \psi^a T^a$ is the adjoint representation of the gauge group and satisfies the Majorana condition,

$$\psi = \psi^C = C\overline{\psi}^T, \quad \overline{\psi} = \overline{\psi}^C = \psi^T\left(-C^{-1}\right). \tag{II.3}$$

The charge conjugation matrix is given as $C = \gamma_0\gamma_2$. We used this condition in the second equality of (II.2). Our γ matrix convention is as follows:

$$\gamma_i = \begin{pmatrix} 0 & -i\sigma^i \\ i\sigma^i & 0 \end{pmatrix}, \quad \gamma_4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma_5 \equiv \gamma_1\gamma_2\gamma_3\gamma_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{II.4}$$

$$\sigma_{\mu\nu} = \frac{1}{2}\left[\gamma_\mu, \gamma_\nu\right], \tag{II.5}$$

$$\epsilon_{1234} = 1. \tag{II.6}$$

Weak coupling perturbation theory is developed by writing the link variable as $U_{x,\mu} = e^{igA_\mu(x+\hat{\mu}/2)}$ and expanding it in terms of gauge coupling g. We adopt a covariant gauge fixing with a gauge parameter α defined by

$$S_{\rm GF} = \sum_n \frac{1}{2\alpha}\left(\nabla_\mu A^a_\mu\left(n + \frac{1}{2}\hat{\mu}\right)\right)^2. \tag{II.7}$$

We set $\alpha = 1$ in this paper. The ghosts do not contribute to our present calculation at one loop level. The gluon propagator can be written as

$$G^{ab}_{\mu\nu}(p) = \frac{1}{4\sin^2 p/2}\left[\delta_{\mu\nu} - (1-\alpha)\frac{4\sin p_\mu/2 \sin p_\nu/2}{4\sin^2 p/2}\right]\delta_{ab}, \tag{II.8}$$

where $\sin^2 p/2 = \sum_\mu \sin^2 p_\mu/2$.

The free gluino propagator is the same as that of the Dirac fermion on lattice,

$$S^{ab}_{\rm F}(p) \equiv \left\langle \psi^a(p)\overline{\psi}^b(-p)\right\rangle = \frac{-i\sum_\mu \gamma_\mu \sin p_\mu + W(p)}{\sum_\mu \sin^2 p_\mu + W(p)^2}\delta_{ab}, \tag{II.9}$$

where

$$W(p) = M + r\sum_\mu\left(1 - \cos p_\mu\right). \tag{II.10}$$

We set the Wilson parameter $r = 1$ in this paper. A peculiar feature of the Majorana fermion is that the propagators which connect two ψ's or two $\overline{\psi}$'s give non zero contribution

$$\left\langle \psi^a(p)\psi^b(-p)\right\rangle = S^{ab}_{\rm F}(p)\left(-C\right), \quad \left\langle \overline{\psi}^a(p)\overline{\psi}^b(-p)\right\rangle = C^{-1}S^{ab}_{\rm F}(p). \tag{II.11}$$

In order to calculate the one loop correction to the SUSY WT identity we need two kinds of gluon-gluino interaction vertex

$$V^{ab,c}_{1\mu}(k,p) = -\frac{1}{2}gf^{abc}\{\gamma_\mu \cos\frac{1}{2}(-k_\mu + p_\mu) - ir\sin\frac{1}{2}(-k_\mu + p_\mu)\}, \tag{II.12}$$

$$V^{ab,cd}_{2\mu\nu}(k,p) = \frac{1}{8}g^2\left(f^{ace}f^{ebd} + f^{ade}f^{ebc}\right)\{i\gamma_\mu \sin\frac{1}{2}(-k_\mu + p_\mu) - r\cos\frac{1}{2}(-k_\mu + p_\mu)\}\delta_{\mu\nu} \tag{II.13}$$

and three gluon self interaction vertex,

$$G^{abc}_{3\mu\nu\rho}(k,l,p) = \frac{1}{3}igf^{abc}\Big(\delta_{\nu\rho}\cos\frac{k_\nu}{2}\sin\frac{1}{2}(p-l)_\mu + \delta_{\rho\mu}\cos\frac{l_\rho}{2}\sin\frac{1}{2}(k-p)_\nu + \delta_{\mu\nu}\cos\frac{p_\mu}{2}\sin\frac{1}{2}(l-k)_\rho\Big). \tag{II.14}$$

Our assignments of momentum and color factors for the vertices are depicted in Fig. 1.

III. AXIAL AND SUSY WARD-TAKAHASHI IDENTITY ON LATTICE

The $U(1)_R$ transformation is given as an axial rotation of the gluino field,

$$\delta\psi = i\alpha\gamma_5\psi, \quad \delta\overline{\psi} = i\alpha\overline{\psi}\gamma_5 \tag{III.1}$$

with a rotation parameter α. The corresponding axial Ward-Takahashi identity is given by

$$\langle(\nabla_\mu j_{5\mu}(n))\,\mathcal{O}\rangle = 2M\,\langle D_A(n)\mathcal{O}\rangle + \langle X_A(n)\mathcal{O}\rangle - \left\langle \frac{\delta\mathcal{O}}{\delta\alpha(n)} \right\rangle, \tag{III.2}$$

where the axial current $j_{5\mu}$ and the symmetry breaking terms D_A, X_A are

$$j_{5\mu}(n) = \mathrm{tr}\left(\overline{\psi}(n)\gamma_\mu\gamma_5 U_\mu(n)\psi(n+\mu)U_\mu^\dagger(n)\right), \tag{III.3}$$

$$D_A(n) = \mathrm{tr}\left(\overline{\psi}(n)\gamma_5\psi(n)\right), \tag{III.4}$$

$$X_A(n) = -r\sum_\mu \mathrm{tr}\Big(\overline{\psi}(n)\gamma_5 U_\mu(n)\psi(n+\mu)U_\mu^\dagger(n) + \overline{\psi}(n)\gamma_5 U_\mu^\dagger(n-\mu)\psi(n-\mu)U_\mu(n-\mu) \\ -2\overline{\psi}(n)\gamma_5\psi(n)\Big). \tag{III.5}$$

∇_μ is a backward derivative, $\mathcal{O}$ is some operator and $\alpha(n)$ is a localized transformation parameter. The trace is taken for the color indices only.

On the other hand there are several choices for the definition of the supertransformation on lattice. The restriction is only to recover the proper form in the continuum limit. Adding to this condition we require the supertransformation to commute with the parity transformation on lattice as in the continuum,

$$\mathcal{P}\psi(\vec{x},t)\mathcal{P}^{-1} = \gamma_0\psi(-\vec{x},t), \quad \mathcal{P}\overline{\psi}(\vec{x},t)\mathcal{P}^{-1} = \overline{\psi}(-\vec{x},t)\gamma_0, \tag{III.6}$$

$$\mathcal{P}U_0(\vec{x},t)\mathcal{P}^{-1} = U_0(-\vec{x},t), \quad \mathcal{P}U_k(\vec{x},t)\mathcal{P}^{-1} = U_k^\dagger(-\vec{x}-\hat{k},t). \tag{III.7}$$

In this paper we adopt the following definition to satisfy the above conditions,

$$\delta_\xi U_\mu(n) = ig\overline{\xi}\gamma_\mu\frac{1}{2}\left(\psi(n)U_\mu(n) + U_\mu(n)\psi(n+\mu)\right), \tag{III.8}$$

$$\delta_\xi U_\mu^\dagger(n) = -ig\overline{\xi}\gamma_\mu\frac{1}{2}\left(U_\mu^\dagger(n)\psi(n) + \psi(n+\mu)U_\mu^\dagger(n)\right), \tag{III.9}$$

$$\delta_\xi\psi(n) = -\frac{1}{2}\sigma_{\mu\nu}\xi P_{\mu\nu}(n), \tag{III.10}$$

$$\delta_\xi\overline{\psi}(n) = \frac{1}{2}\overline{\xi}\sigma_{\mu\nu}P_{\mu\nu}(n), \tag{III.11}$$

where ξ, $\overline{\xi}$ are fermionic transformation parameter satisfying the Majorana condition. For the field strength $P_{\mu\nu}$ we employ the definition with clover plaquette,

$$P_{\mu\nu}(n) = \frac{1}{4}\sum_{i=1}^{4}\frac{1}{2ig}\left(U_{i\mu\nu}(n) - U_{i\mu\nu}^\dagger(n)\right), \tag{III.12}$$

$$U_{1\mu\nu}(n) = U_\mu(n)U_\nu(n+\hat{\mu})U_\mu^\dagger(n+\hat{\nu})U_\nu^\dagger(n), \tag{III.13}$$

$$U_{2\mu\nu}(n) = U_\nu(n)U_\mu^\dagger(n-\hat{\mu}+\hat{\nu})U_\nu^\dagger(n-\hat{\mu})U_\mu(n-\hat{\mu}), \tag{III.14}$$

$$U_{3\mu\nu}(n) = U_\mu^\dagger(n-\hat{\mu})U_\nu^\dagger(n-\hat{\mu}-\hat{\nu})U_\mu(n-\hat{\mu}-\hat{\nu})U_\nu(n-\hat{\nu}), \tag{III.15}$$

$$U_{4\mu\nu}(n) = U_\nu^\dagger(n-\hat{\nu})U_\mu(n-\hat{\nu})U_\nu(n+\hat{\mu}-\hat{\nu})U_\mu^\dagger(n). \tag{III.16}$$

This definition is slightly different from the original one [5].

By transforming the vacuum expectation value of some operator

$$\langle \mathcal{O} \rangle = \int dU d\psi \mathcal{O} e^{-S_{\text{gluon}} - S_{\text{gluino}}} \tag{III.17}$$

with a localized transformation parameter we find the SUSY WT identity on lattice,

$$\langle (\nabla_\mu S_\mu(n))\, \mathcal{O} \rangle \ = M \langle D_S(n) \mathcal{O} \rangle + \langle X_S(n) \mathcal{O} \rangle - \left\langle \frac{\delta \mathcal{O}}{\delta \bar{\xi}(n)} \right\rangle, \tag{III.18}$$

where the supercurrent S_μ and the gluino mass term D_S become

$$S_\mu(n) = -\frac{1}{2} \sum_{\rho\sigma} \sigma_{\rho\sigma} \gamma_\mu \mathrm{tr}\Big(P_{\rho\sigma}(n) U_\mu(n) \psi(n+\hat{\mu}) U_\mu^\dagger(n) + P_{\rho\sigma}(n+\mu) U_\mu^\dagger(n) \psi(n) U_\mu(n) \Big), \tag{III.19}$$

$$D_S(n) = \sum_{\rho\sigma} \sigma_{\rho\sigma} \mathrm{tr}\left(P_{\rho\sigma}(n) \psi(n) \right). \tag{III.20}$$

The explicit SUSY breaking term X_S is given by a sum of four terms

$$X_S(n) = X_S^{(1)}(n) + X_S^{(2)}(n) + X_S^{(3)}(n) + X_S^{(4)}(n) \tag{III.21}$$

with

$$\begin{aligned} X_S^{(1)}(n) = \sum_{\mu\rho\sigma} r \sigma_{\rho\sigma} \mathrm{tr}\Big[P_{\rho\sigma}(n) \Big(\psi(n) &- \frac{1}{2} \left(U_\mu(n) \psi(n+\mu) U_\mu^\dagger(n) \right) \\ &- \frac{1}{2} \left(U_\mu^\dagger(n-\mu) \right) \psi(n-\mu) U_\mu(n-\mu) \Big) \Big], \end{aligned} \tag{III.22}$$

$$\begin{aligned} X_S^{(2)}(n) = \sum_{\mu\nu\rho\sigma} \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \gamma_\nu \gamma_5 \mathrm{tr}\Big(& U_\mu^\dagger(n) \psi(n) U_\mu(n) P_{\rho\sigma}(n+\mu) \\ & - U_\mu(n-\mu) \psi(n) U_\mu^\dagger(n-\mu) P_{\rho\sigma}(n-\mu) \Big), \end{aligned} \tag{III.23}$$

$$\begin{aligned} X_S^{(3)}(n) = \sum_{\mu\nu} \gamma_\nu \mathrm{tr}\Big(& U_\mu(n-\mu) \psi(n) U_\mu^\dagger(n-\mu) H_{\mu\nu}(n-\mu) + U_\mu^\dagger(n) \psi(n) U_\mu(n) H_{\mu\nu}(n+\mu) \\ & - 2\psi(n) H_{\mu\nu}(n) \Big), \end{aligned} \tag{III.24}$$

$$\begin{aligned} X_S^{(4)}(n) = -\frac{1}{2} ig \sum_\mu \Big[& \left(\gamma_\mu \psi^a(n) \right) \overline{\psi}^b(n) \left(-r + \gamma_\mu \right) \psi^c(n+\mu) \mathrm{tr}\left(\left[T^a, T^b \right] U_\mu(n) T^c U_\mu^\dagger(n) \right) \\ & + \left(\gamma_\mu \psi^a(n) \right) \overline{\psi}^b(n) \left(r + \gamma_\mu \right) \psi^c(n-\mu) \mathrm{tr}\left(\left[T^a, T^b \right] U_\mu^\dagger(n-\mu) T^c U_\mu(n-\mu) \right) \Big], \end{aligned} \tag{III.25}$$

where $H_{\mu\nu}$ is given by a subtraction between the clover leafs,

$$\begin{aligned} H_{\mu\nu}(n) = \frac{1}{8ig} \Big(& \left(U_{1,\mu\nu}(n) - U_{1,\mu\nu}^\dagger(n) \right) + \left(U_{4,\mu\nu}(n) - U_{4,\mu\nu}^\dagger(n) \right) \\ & - \left(U_{2,\mu\nu}(n) - U_{2,\mu\nu}^\dagger(n) \right) - \left(U_{3,\mu\nu}(n) - U_{3,\mu\nu}^\dagger(n) \right) \Big). \end{aligned} \tag{III.26}$$

Here we notice that $X_S^{(2)}$, $X_S^{(3)}$ come from the pure gauge part and $X_S^{(1)}$, $X_S^{(4)}$ are originated from the gluino action. Especially $X_S^{(1)}$ is given by transforming the gluino field in the Wilson term.

At tree level the breaking terms due to the Wilson fermion $X_S^{(1)}$, $X_S^{(4)}$ represent the $\mathcal{O}(a)$ irrelevant operator and those from the plaquette action $X_S^{(2)}$, $X_S^{(3)}$ represent the $\mathcal{O}(a^2)$ operator. The SUSY WT identity is recovered in the continuum with the gluino mass set to zero since all the irrelevant operators vanish and the lattice supercurrent $S_\mu(n)$ gives the continuum form

$$S_\mu(n) \to -\sum_{\rho\sigma} \sigma_{\rho\sigma}\gamma_\mu \mathrm{tr}\left(F_{\rho\sigma}\psi\right). \tag{III.27}$$

However at one loop order every term gives the finite contribution and the symmetry restoration becomes nontrivial.

Derivation of the Feynman rules for the supercurrent and breaking terms is straightforward but tedious task and the results are very complicated. We omitted them in this paper.

IV. ONE LOOP CORRECTION TO AXIAL WARD-TAKAHASHI IDENTITY

In this section we calculate the one loop correction to the axial Ward-Takahashi identity. We consider the WT identity with $\mathcal{O} = \psi(y)\overline{\psi}(z)$:

$$\begin{aligned} 0 = &\left\langle \left(\nabla_\mu j_{5\mu}(n)\right)\psi(y)\overline{\psi}(z)\right\rangle - 2M\left\langle D_A(n)\psi(y)\overline{\psi}(z)\right\rangle - \left\langle X_A(n)\psi(y)\overline{\psi}(z)\right\rangle \\ &+\delta_{n,y}\gamma_5\left\langle \psi(n)\overline{\psi}(z)\right\rangle + \delta_{n,z}\left\langle \psi(y)\overline{\psi}(n)\right\rangle\gamma_5. \end{aligned} \tag{IV.1}$$

We calculate the quantum correction to each Green functions in the identity. Although our fermion is Majorana the one loop correction becomes the same as that for the Dirac fermion system [14] except for the color factor. One loop contributions to the Green functions $\left\langle \left(\nabla_\mu j_{5\mu}\right)\psi\overline{\psi}\right\rangle$ and $\left\langle X_A\psi\overline{\psi}\right\rangle$ are given by the four diagrams in Fig. 2. One loop level full Green functions become

$$\left\langle \left(\nabla_\mu j_{5\mu}\right)\psi(k)\overline{\psi}(p)\right\rangle^{\mathrm{full}} = \frac{Z_2}{-i\not{k} + Z_m^{-1}M} i(k+p)_\mu T_A\gamma_\mu\gamma_5 \frac{Z_2}{i\not{p} + Z_m^{-1}M}, \tag{IV.2}$$

$$\left\langle X_A\psi(k)\overline{\psi}(p)\right\rangle^{\mathrm{full}} = \frac{Z_2}{-i\not{k} + Z_m^{-1}M}\left(i(k+p)_\mu X_a\gamma_\mu\gamma_5 + (X_m M + X_0)\gamma_5\right)\frac{Z_2}{i\not{p} + Z_m^{-1}M}. \tag{IV.3}$$

One loop correction to $\left\langle D_A\psi\overline{\psi}\right\rangle$ is given by the first diagram in Fig. 2,

$$\left\langle D_A\psi(k)\overline{\psi}(p)\right\rangle^{\mathrm{full}} = \frac{Z_2}{-i\not{k} + Z_m^{-1}M} T_P\gamma_5 \frac{Z_2}{i\not{p} + Z_m^{-1}M}. \tag{IV.4}$$

The vertex corrections T_A and T_P depend on the gluino external momentum and mass and they have infra-red divergence at $k = p = M = 0$ in general. In order to regularize the infra-red singularity we adopt the following subtraction scheme,

$$T_A(k,p,M) = \left(T_A(k,p,M) - T_A^{\mathrm{cont.}}(k,p,M)\right) + T_A^{\mathrm{cont.}}(k,p,M), \tag{IV.5}$$

where $T_A^{\text{cont.}}$ is a vertex correction given by integrating the continuum Feynman rule with lattice loop momentum between $-\pi/a$ and π/a. Since $T_A^{\text{cont.}}$ has the same IR singularity, the IR divergence is subtracted in the first term of (IV.5). We evaluate it by a Taylor expansion around $(k, p, M) = (0, 0, 0)$. The second term should be calculated analytically keeping the IR regulator finite. In this paper we introduce a gluon mass λ into the gluon propagator inside the loop as an IR regularization [1]. We can evaluate $T_A^{\text{cont.}}$ quite simply with a Taylor expansion in terms of (k, p, M) keeping λ finite,

$$T_A^{\text{cont.}}(k, p, M; \lambda) = T_A^{\text{cont.}}(0; \lambda) + k_\mu \frac{\partial T_A^{\text{cont.}}(0; \lambda)}{\partial k_\mu} + \cdots, \tag{IV.6}$$

where $T_A^{\text{cont.}}(0; \lambda)$ contribute to the renormalization of the operator and the remaining terms are $\mathcal{O}(a)$ errors.

In this scheme the vertex corrections are given as follows

$$T_A = 1 + \frac{g^2}{16\pi^2} N_c \left[-\log \frac{(\lambda a)^2}{\pi^2} - 6.977 \right], \tag{IV.7}$$

$$T_P = 1 + \frac{g^2}{16\pi^2} N_c \left[-4\log \frac{(\lambda a)^2}{\pi^2} + 2.585 \right], \tag{IV.8}$$

$$X_a = \frac{g^2}{16\pi^2} N_c (8.664), \tag{IV.9}$$

$$X_m = \frac{g^2}{16\pi^2} N_c (-19.285), \tag{IV.10}$$

$$X_0 = \frac{g^2}{16\pi^2} N_c (102.8694). \tag{IV.11}$$

The gluino wave function and mass renormalization factors are evaluated by the quantum correction to the gluino propagator,

$$Z_2 = 1 + \frac{g^2}{16\pi^2} N_c \left[\log \frac{(\lambda a)^2}{\pi^2} + 15.641 \right], \tag{IV.12}$$

$$Z_m = 1 + \frac{g^2}{16\pi^2} N_c \left[3\log \frac{(\lambda a)^2}{\pi^2} - 8.584 \right]. \tag{IV.13}$$

The renormalization of operators is given to keep the proper form of the axial WT identity,

$$G_A = \left\langle (\nabla_\mu j_{5\mu})\, \psi(k)\overline{\psi}(p) \right\rangle^{\text{full}} - 2M \left\langle D_A \psi(k)\overline{\psi}(p) \right\rangle^{\text{full}} - \left\langle X_A \psi(p)\overline{\psi}(p) \right\rangle^{\text{full}}$$

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[1] Strictly speaking this IR regularization procedure cannot be applied to all the quantum corrections in the SUSY WT identity because there is a gluon propagator in the external line and its wave function renormalization is required. The IR divergence of the gluino and ghost loops in the gluon polarization cannot be regularized. However we proceed our calculation with this scheme since we are only interested in the one loop vertex correction whose IR divergence is treatable in this method.

$$= \frac{Z_2}{-i\not{k} + Z_m^{-1} M}\left[i(k+p)_\mu \left(T_A - X_a\right)\gamma_\mu\gamma_5 - 2\left(M\left(T_P + \frac{X_m}{2}\right) + \frac{X_0}{2}\right)\gamma_5\right]\frac{Z_2}{i\not{p} + Z_m^{-1} M} \tag{IV.14}$$

$$= Z_2\left(Z_A\left\langle\left(\partial_\mu j_{5\mu}\right)\psi(k)\overline{\psi}(p)\right\rangle_R - 2M_R\left\langle D_A\psi(k)\overline{\psi}(p)\right\rangle_R\right), \tag{IV.15}$$

where suffix R means renormalized quantity. The gluino mass is renormalized as

$$M_R = Z_m^{-1}\left(M - \Sigma_0\right), \tag{IV.16}$$

where Σ_0 is an additive mass correction

$$\Sigma_0 = -\frac{X_0}{2} = \frac{g^2}{16\pi^2} N_c(-51.4347), \tag{IV.17}$$

which gives the critical hopping parameter corresponding to the chiral symmetric point

$$K_c = \frac{1}{8}\left(1 - \frac{\Sigma_0}{4}\right). \tag{IV.18}$$

The multiplicative mass renormalization factor evaluated from the WT identity agrees with that from the gluino propagator

$$Z_m^{-1} = Z_2\left(T_P + \frac{X_m}{2}\right) = 1 - \frac{g^2}{16\pi^2} N_c\left[3\log\frac{(\lambda a)^2}{\pi^2} - 8.584\right]. \tag{IV.19}$$

The renormalization factors of the axial current becomes unity in order to keep the axial WT identity

$$Z_A = Z_2\left(T_A - X_a\right) = 1.000. \tag{IV.20}$$

Numerical errors of the finite parts in this section are in the last digit written.

V. ONE LOOP CORRECTION TO SUSY WARD-TAKAHASHI IDENTITY

We consider the following SUSY WT identity with $\mathcal{O} = A_\alpha(y)\psi(z)$:

$$\begin{aligned} 0 = &\left\langle\left(\nabla_\mu S_\mu(n)\right)A_\alpha(y)\overline{\psi}(z)\right\rangle - M\left\langle D_S(n)A_\alpha(y)\overline{\psi}(z)\right\rangle - \left\langle X_S(n)A_\alpha(y)\overline{\psi}(z)\right\rangle \\ &+\delta_{n,y}\gamma_\alpha\left\langle\psi(y)\overline{\psi}(z)\right\rangle + \delta_{n,z}\frac{1}{2}\sigma_{\rho\sigma}\left\langle A_\alpha(y)P_{\rho\sigma}(z)\right\rangle. \end{aligned} \tag{V.1}$$

Although these Green functions are gauge variant we can derive gauge independent quantities such as additive mass correction correctly. One loop corrections to the Green functions $\left\langle\left(\nabla_\mu S_\mu\right)A_\alpha\overline{\psi}\right\rangle$, $\left\langle D_S A_\alpha\overline{\psi}\right\rangle$ and $\left\langle X_S^{(i)} A_\alpha\overline{\psi}\right\rangle$ with $i = 1, 2, 3$ are given by the diagrams in Fig. 3. The one loop correction to the explicit breaking term with $i = 4$ is given by the two diagrams in Fig. 4.

As was discussed in the previous section we introduce the gluon mass λ here again in order to settle the IR singularity at vanishing external momentum and gluino mass. All the IR divergence in the one loop vertex correction appears from the first four diagrams of

Fig. 3 and can be regularized in this scheme. The fermion loops in Fig. 4 do not have any IR divergence.

We calculate the one loop contribution imposing the on-shell condition to the external gluino momentum,

$$\Gamma(i\not{p} + M_R) = 0, \tag{V.2}$$

where Γ represents some operator vertex. This is applicable to the bare mass inside the one loop correction by making use of the relation $g^2 M = g^2 M_R + \mathcal{O}(g^4)$. The one loop correction to the Green functions $\left\langle (\nabla_\mu S_\mu)\, A_\alpha \overline{\psi} \right\rangle$ and $\left\langle D_S A_\alpha \overline{\psi} \right\rangle$ are

$$\left\langle (\nabla_\mu S_\mu)\, A_\alpha(k)\overline{\psi}(p) \right\rangle_1 = \frac{1}{k^2}(k_\mu + p_\mu)\Big(k_\nu \sigma_{\nu\alpha}\gamma_\mu T_S^{(S)} + (\delta_{\mu\alpha}\not{k} - k_\mu \gamma_\alpha)\, T_T^{(S)}\Big)\frac{1}{i\not{p} + M}, \tag{V.3}$$

$$\left\langle D_S A_\alpha(k)\overline{\psi}(p) \right\rangle_1 = \frac{1}{k^2}\left(iMk_\mu \sigma_{\mu\alpha} T_D^{(D)} + p_\alpha \not{p} T_{GP}^{(D)} + p^2 \gamma_\alpha T_{GM}^{(D)}\right)\frac{1}{i\not{p} + M}, \tag{V.4}$$

where

$$T_S^{(S)} = \frac{g^2}{16\pi^2} N_c \left(-27.874(1) + \frac{2\pi^2}{N_c^2}\right), \tag{V.5}$$

$$T_T^{(S)} = \frac{g^2}{16\pi^2} N_c \left(6.372(1)\right), \tag{V.6}$$

$$T_D^{(D)} = \frac{g^2}{16\pi^2} N_c \left(-3\ln\frac{(\lambda a)^2}{\pi^2} - 23.6453(7) + \frac{2\pi^2}{N_c^2}\right), \tag{V.7}$$

$$T_{GP}^{(D)} = \frac{g^2}{16\pi^2} N_c \left(1.000(1)\right), \tag{V.8}$$

$$T_{GM}^{(D)} = \frac{g^2}{16\pi^2} N_c \left(-1.000(1)\right). \tag{V.9}$$

The finite term proportional to $1/N_c$ emerges from the last tadpole diagram in Fig. 3. In the above the finite part of the loop correction is evaluated by performing the loop integrals with the Monte Carlo routine VEGAS in double precision. We employ 20 sets of 10^6 points for integration. Errors are estimated from variation of integrated values over the sets. We eliminated the terms when their coefficients become smaller than the numerical errors. Here we remind that the logarithmic divergence appears only in the term concerning to the mass renormalization.

The one loop contribution to the explicit breaking term $\left\langle X_S A_\alpha \overline{\psi} \right\rangle$ is given by

$$\begin{aligned}\left\langle X_S A_\alpha(k)\overline{\psi}(p) \right\rangle_1 = \frac{1}{k^2}\Big[&(k+p)_\mu \left(k_\nu \sigma_{\nu\alpha}\gamma_\mu T_S^{(X)} + (\delta_{\mu\alpha}\not{k} - k_\mu\gamma_\alpha)\, T_T^{(X)}\right) \\ &+ i k_\mu \sigma_{\mu\alpha}\left(T_D^{\text{add.}} + M T_D^{(X)}\right) \\ &+ k_\alpha \not{p} T_{GK}^{(X)} + p^2 \gamma_\alpha T_{GM}^{(X)}\Big]\frac{1}{i\not{p} + M},\end{aligned} \tag{V.10}$$

where

$$T_S^{(X)} = \frac{g^2}{16\pi^2} N_c \left(-7.775(1)\right), \tag{V.11}$$

$$T_T^{(X)} = \frac{g^2}{16\pi^2} N_c \left(3.3716(7)\right), \tag{V.12}$$

$$T_D^{\text{add.}} = \frac{g^2}{16\pi^2} N_c (51.4345(5)), \tag{V.13}$$

$$T_D^{(X)} = \frac{g^2}{16\pi^2} N_c \left(11.130(1)\right), \tag{V.14}$$

$$T_{GK}^{(X)} = \frac{g^2}{16\pi^2} N_c \left(-2.000(1)\right), \tag{V.15}$$

$$T_{GM}^{(X)} = \frac{g^2}{16\pi^2} N_c \left(-1.842(2)\right). \tag{V.16}$$

Here we should notice that the one loop correction to the SUSY explicit breaking term X_S produces an additive mass correction given by (V.13). This additive correction coincide with that from the axial Ward-Takahashi identity (IV.17) within a numerical error. This fact confirms the prediction of Ref. [5]. We have a comment on the origin of this additive correction. The SUSY explicit breaking term X_S can be classified into four parts $X_S^{(i)}$. $X_S^{(1)}$ is given by supertransforming the gluino fields of the Wilson term, $X_S^{(4)}$ appears from the gluino action by transforming the link variable in the covariant derivative. $X_S^{(2)}$, $X_S^{(3)}$ are originated from the gluon plaquette action. Since the additive mass in the axial WT identity (IV.17) is due to the axial symmetry breaking term given by rotating the Wilson term, it might have been expected that the additive correction in the SUSY WT identity comes only from $X_S^{(1)}$, $X_S^{(4)}$ which are directly related to the Wilson term. However $X_S^{(1)}$, $X_S^{(4)}$ produces only 80% of $T_D^{\text{add.}}$ and remaining 20% is a contribution from $X_S^{(2)}$, $X_S^{(3)}$. Contributions from each $X_S^{(i)}$ is given in table I. The Wilson parameter dependence of $T_D^{\text{add.}}$ is given nontrivially inside the diagram multiplying $X_S^{(2)}$, $X_S^{(3)}$ with the Wilson parameter in the gluino propagator and the interaction vertex. We depicted the r dependence of Σ_0 and $T_D^{\text{add.}}$ in table II, which are in good agreement for every r within a numerical error.

Summing up all the contributions we can investigate the mixing behavior.

$$\begin{aligned} G_S^{(1)} &= \left\langle \left(\nabla_\mu S_\mu\right) A_\alpha(k) \overline{\psi}(p) \right\rangle_1 - M \left\langle D_S A_\alpha(k) \overline{\psi}(p) \right\rangle_1 - \left\langle X_S A_\alpha(k) \overline{\psi}(p) \right\rangle_1 \\ &= \frac{1}{k^2} \Big[(k_\mu + p_\mu) \left(k_\nu \sigma_{\nu\alpha} \gamma_\mu T_S^{(1)} + \left(\delta_{\mu\alpha} \not{k} - k_\mu \gamma_\alpha\right) T_T^{(1)} \right) \\ &\qquad - i k_\mu \sigma_{\mu\alpha} \left(M T_D^{(1)} + T_D^{\text{add.}} \right) \\ &\qquad - k_\alpha \not{p} T_{GK} - p_\alpha \not{p} T_{GP} - p^2 \gamma_\alpha T_{GM} \Big] \frac{1}{i\not{p} + M}, \end{aligned} \tag{V.17}$$

where

$$T_S^{(1)} = T_S^{(S)} - T_S^{(X)} = \frac{g^2}{16\pi^2} N_c \left(-20.099(2) + \frac{2\pi^2}{N_c^2} \right), \tag{V.18}$$

$$T_T^{(1)} = T_T^{(S)} - T_T^{(X)} = \frac{g^2}{16\pi^2} N_c \left(3.000(2)\right), \tag{V.19}$$

$$T_D^{(1)} = T_D^{(D)} + T_D^{(X)} = \frac{g^2}{16\pi^2} N_c \left(-3 \ln \frac{(\lambda a)^2}{\pi^2} - 12.515(2) + \frac{2\pi^2}{N_c^2} \right), \tag{V.20}$$

$$T_{GK} = T_{GK}^{(X)} = \frac{g^2}{16\pi^2} N_c \left(-2.000(1)\right), \tag{V.21}$$

$$T_{GP} = T_{GP}^{(D)} = \frac{g^2}{16\pi^2} N_c \left(1.000(1)\right), \tag{V.22}$$

$$T_{GM} = T_{GM}^{(D)} + T_{GM}^{(X)} = \frac{g^2}{16\pi^2} N_c \left(-2.842(3)\right). \tag{V.23}$$

Now we consider the mixing property of the operators by making use of the continuum form of the Green functions

$$\left\langle (\partial_\mu S_\mu) A_\alpha(k) \overline{\psi}(p) \right\rangle = \frac{1}{k^2} (k_\mu + p_\mu) k_\nu \sigma_{\nu\alpha} \gamma_\mu \frac{1}{i\not{p} + M}, \tag{V.24}$$

$$\left\langle (\partial_\mu T_\mu) A_\alpha(k) \overline{\psi}(p) \right\rangle = \frac{1}{k^2} (k_\mu + p_\mu) \left(\delta_{\mu\alpha} \not{k} - k_\mu \gamma_\alpha\right) \frac{1}{i\not{p} + M}, \tag{V.25}$$

$$\left\langle D_S A_\alpha(k) \overline{\psi}(p) \right\rangle = \frac{1}{k^2} M i k_\nu \sigma_{\nu\alpha} \frac{1}{i\not{p} + M}, \tag{V.26}$$

$$\left\langle \left(\partial_\mu A_\mu \not{\partial} \psi\right) A_\alpha(k) \overline{\psi}(p) \right\rangle = -\frac{1}{k^2} k_\alpha \not{p} \frac{1}{i\not{p} + M}, \tag{V.27}$$

$$\left\langle \left(A_\mu \partial_\mu \not{\partial} \psi\right) A_\alpha(k) \overline{\psi}(p) \right\rangle = -\frac{1}{k^2} p_\alpha \not{p} \frac{1}{i\not{p} + M}, \tag{V.28}$$

$$\left\langle \left(\not{A} \partial_\mu \partial_\mu \psi\right) A_\alpha(k) \overline{\psi}(p) \right\rangle = -\frac{1}{k^2} p^2 \gamma_\alpha \frac{1}{i\not{p} + M}. \tag{V.29}$$

Here we introduce a gauge invariant fermionic current with dimension 7/2

$$T_\mu(n) = 2\mathrm{tr}\left(P_{\mu\nu}(n) \gamma_\nu \psi(n)\right). \tag{V.30}$$

We can easily see that the first term in (V.17) with $T_S^{(1)}$ contribute to the multiplicative normalization factor of the supercurrent. The second term with $T_T^{(1)}$ represents the mixing with a gauge invariant current T_μ as was discussed in Ref. [5]. Mixing with T_μ is also reported in the continuum theory with the dimensional regularization [15]. This term is related to the gamma-trace anomaly corresponding to the super conformal symmetry breaking. Its coefficient $T_T^{(1)}$ is identical with the one loop level β-function of the $N = 1$ SYM,

$$T_T^{(1)} = \frac{g^2}{16\pi^2} 3N_c = -\frac{\beta_{1-\mathrm{loop}}}{g} \tag{V.31}$$

as was required for the gamma-trace anomaly [12]. The third term gives the additive and multiplicative mass correction. The remaining three terms are mixing with gauge variant operators. This mixing is because we adopted gauge variant operator $\mathcal{O} = A_\alpha(y)\psi(z)$ in (V.1) and fixed gauge in perturbative calculation. However these extra mixings disappear if we impose on-shell condition to the gluino momentum and set the renormalized gluino mass to zero

$$i\not{p} = -M_R = 0. \tag{V.32}$$

Hereafter we assume the existence of a consistent IR regularization scheme with the gluon mass procedure for the gluon wave function. If the gluon wave function renormalization

factor Z_3 is derived in a well defined manner the one loop level full Green function can be given

$$\begin{aligned}
G_S &= \left\langle (\nabla_\mu S_\mu)\, A_\alpha(k)\overline{\psi}(p)\right\rangle^{\text{full}} - M\left\langle D_S A_\alpha(y)\overline{\psi}(z)\right\rangle^{\text{full}} - \left\langle X_S A_\alpha(k)\overline{\psi}(p)\right\rangle^{\text{full}} \\
&= Z_S\sqrt{Z_3Z_2}\left\langle (\nabla_\mu S_\mu)\, A_\alpha(k)\overline{\psi}(p)\right\rangle_R \\
&\quad - Z_D\sqrt{Z_3Z_2}\left(M + T_D^{\text{add.}}\right)\left\langle D_S A_\alpha(k)\overline{\psi}(p)\right\rangle_R \\
&\quad + T_T^{(1)}\left\langle (\nabla_\mu T_\mu)\, A_\alpha(k)\overline{\psi}(p)\right\rangle_R \\
&\quad + T_{GK}\left\langle (\partial_\mu A_\mu \partial\!\!\!/ \psi)\, A_\alpha(k)\overline{\psi}(p)\right\rangle_R \\
&\quad + T_{GP}\left\langle (A_\mu \partial_\mu \partial\!\!\!/ \psi)\, A_\alpha(k)\overline{\psi}(p)\right\rangle_R \\
&\quad + T_{GM}\left\langle (A\!\!\!/ \partial_\mu\partial_\mu \psi)\, A_\alpha(k)\overline{\psi}(p)\right\rangle_R ,
\end{aligned} \tag{V.33}$$

where Z_2 is the gluino wave function renormalization factor (IV.12). The renormalization factors are given as

$$Z_S = \sqrt{Z_3Z_2}\left(1 + T_S^{(1)}\right), \quad Z_D = \sqrt{Z_3Z_2}\left(1 + T_D^{(1)}\right). \tag{V.34}$$

The logarithmic divergences in the gluon and gluino wave function renormalization factor are canceled with the multiplication Z_3Z_2 as is easily confirmed in the continuum with dimensional regularization and the supercurrent renormalization factor Z_S remains finite. The renormalization of the gluino mass is

$$\widetilde{M}_R = Z_D\left(M + T_D^{\text{add.}}\right), \tag{V.35}$$

which gives the same critical mass as in (IV.17).

VI. CONCLUSION

In this article we regularize the supersymmetric Yang-Mills theory on lattice with the Wilson plaquette action for gluon and the Wilson fermion for gluino. In this regularization the supersymmetry and axial $U(1)_R$ symmetry of the continuum SYM theory is broken explicitly. However both of the symmetries can be recovered in the continuum by fine tuning the mass parameter.

In order to see this restoration process we calculated the one loop correction to the SUSY Ward-Takahashi identity perturbatively. It is shown that the additive mass correction (the critical mass) given by the SUSY Ward-Takahashi identity coincides with that from the axial WT identity. This means the SUSY and the $U(1)_R$ symmetry can be restored simultaneously in the continuum limit with a single fine tuning of the gluino mass. Since tuning to the chiral symmetric point is a well known subject in computer simulation, there would have been no technical difficulty to deal with the SYM system on lattice even nonperturbatively if the axial $U(1)_R$ symmetry had no anomaly. The chiral symmetric point cannot be given by the vanishing pion mass for the anomalous $U(1)_R$ symmetry of SYM and an alternative method is needed. Application of the vacuum degeneracy of residual Z_{2N_c} symmetry due to gluino condensation seems to be hopeful [10].

The supercurrent on lattice mixes nontrivially with T_μ. If we can extract this gamma-trace anomaly part nonperturbatively from the SUSY WT identity we may be able to evaluate the exact β-function of the $N = 1$ SYM theory.

A peculiar point in our calculation is that the SUSY breaking term given by supertransforming the plaquette action also contribute to the additive mass correction. Therefore when we consider to use the domain-wall fermion as a gluino part, it is nontrivial to see disappearance of the additive mass correction. Because the domain-wall fermion system contains the Wilson term in its action before integrating out the unphysical heavy modes. The negative unity Wilson parameter remains in the gluino-gluon interaction vertex. This is a fascinating future problem.

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REFERENCES

[1] For a review, see: K. Intriligator and N. Seiberg, Nucl. Phys. **B45** (Proc. Suppl.) (1996) 1 and references therein.

[2] T. Aoyama and Y. Kikukawa, Phys. Rev. **D59** (1999) 054507.

[3] W. Bietenholz, Mod. Phys. Lett. **A14** (1999) 51.

[4] H. So and N. Ukita , hep-lat/9812002.

[5] G. Curci and G. Veneziano, Nucl. Phys. **B292** (1987) 555.

[6] I. Montvay, Nucl. Phys. **B466** (1996) 259.

[7] A. Donini and M. Guagnelli, Phys. Lett. **B383** (1996) 301.

[8] G. Koutsoubas and I. Montvay, Phys. Lett. **B398** (1997) 130.

[9] A. Donini, M. Guagnelli, P. Hernandez and A. Vladikas, Nucl. Phys. **B523** (1998) 529.

[10] DESY-Münster Collaboration, R. Kirchner, S. Luckmann, I. Montvay, K. Spanderen and J. Westphalen, Nucl. Phys. B (Proc. Suppl.) **73** (1999) 828; Phys. Lett. **B446** (1999) 209.

[11] DESY-Munster Collaboration, I. Campos, A. Feo, R. Kirchner, S. Luckmann, I. Montvay, G. Münster, K. Spanderen, J. Westphalen, hep-lat/9903014.

[12] G. Veneziano and S. Yankielowicz, Phys. Lett. **B113** (1982) 213.

[13] G.R. Farrar, G. Gabadadze, M. Schwetz, Phys. Rev. **D58** (1998) 015009.

[14] M. Bochicchio, L. Maiani, G. Martinelli, G. Rossi and M. Testa, Nucl. Phys. **B262** (1985) 331.

[15] L. F. Abbott, M. T. Grisaru and H. J. Schnitzer, Phys. Rev. **D16** (1977) 2995.

TABLES

TABLE I. Contribution to additive mass correction from each $X_S^{(i)}$ at $r = 1$. 80% of the contribution is given by $X_S^{(1)}$ and $X_S^{(4)}$.

X_S^i	$X_S^{(1)}$	$X_S^{(2)}$	$X_S^{(3)}$	$X_S^{(4)}$
$T_D^{\mathrm{add.}}(i)$	42.2083(5)	7.32191(2)	2.85525(4)	−0.95093(2)

TABLE II. Wilson parameter dependence of the additive mass correction from axial and SUSY WT identity. Both results are in good agreement for every r within a numerical error.

r	0	0.2	0.4	0.6	0.8	1.0
$T_D^{\mathrm{add.}}$	0	19.793(8)	30.707(8)	38.286(7)	44.964(8)	51.4345(5)
Σ_0	0	19.791(1)	30.695(2)	38.283(3)	44.960(4)	51.4346(1)

FIGURES

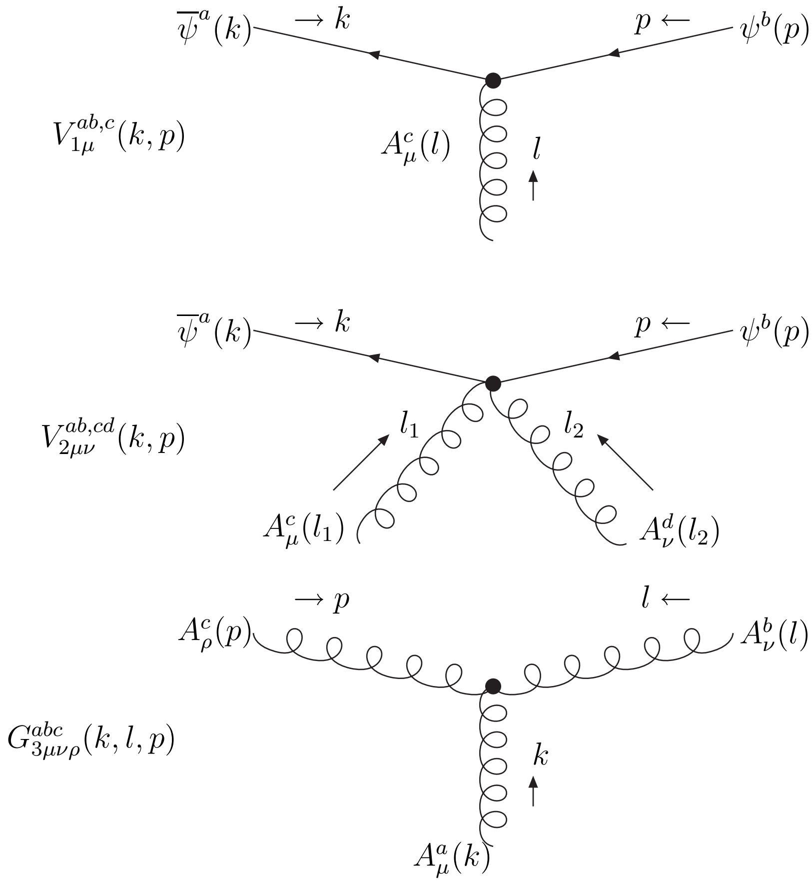
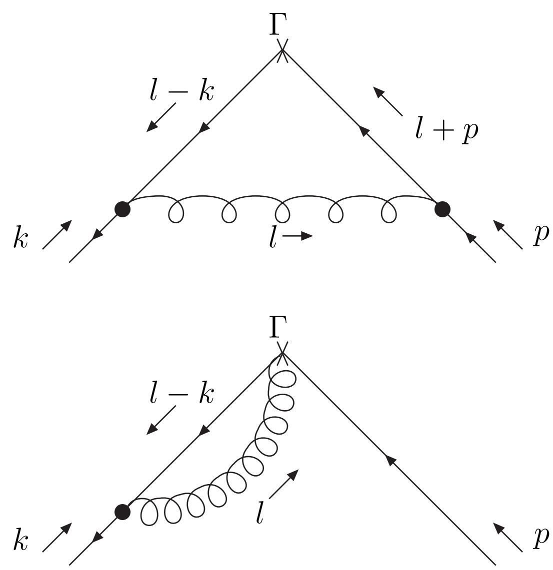


FIG. 1. Quark-gluon vertices needed for our one-loop calculations.



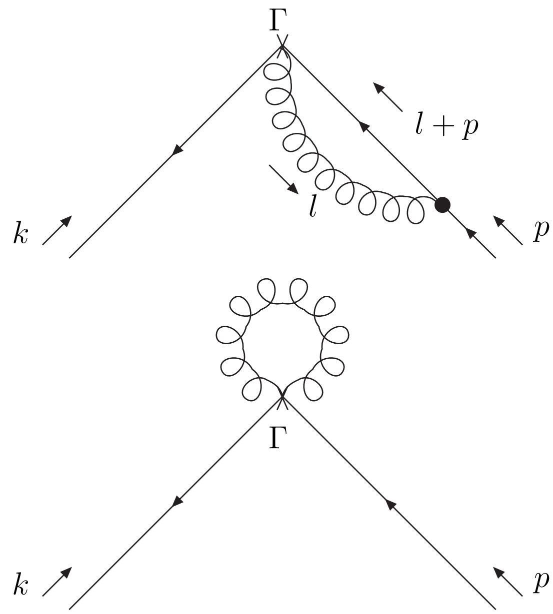
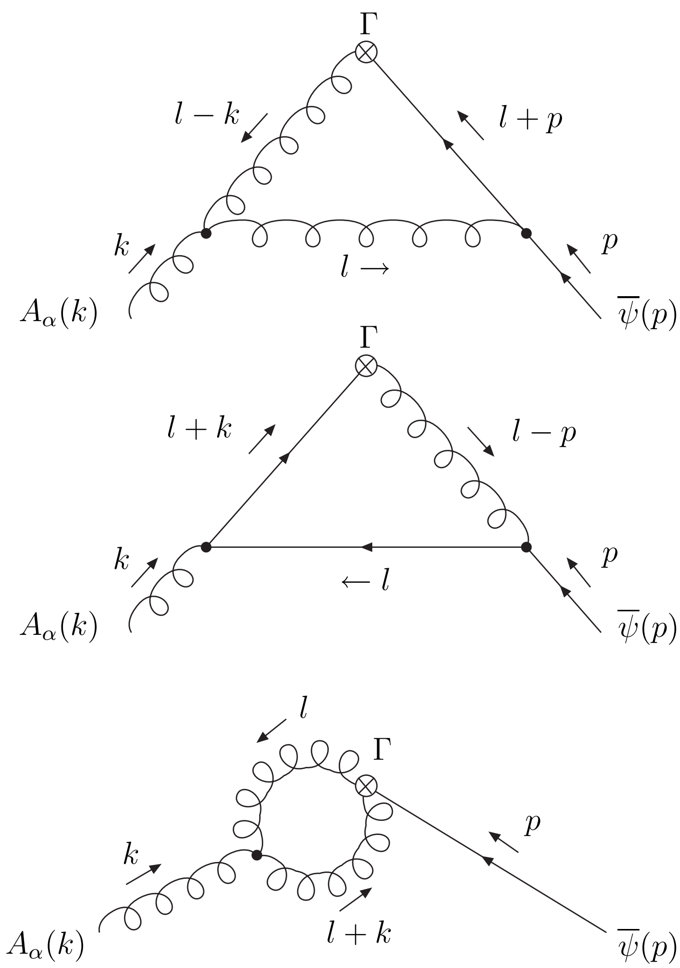


FIG. 2. One loop diagrams which contribute to the operator vertex correction in the axial Ward-Takahashi identity. $\Gamma = j_{5\mu}, D_A, X_A$ for the first diagram and $\Gamma = j_{5\mu}, X_A$ for the remaining three.



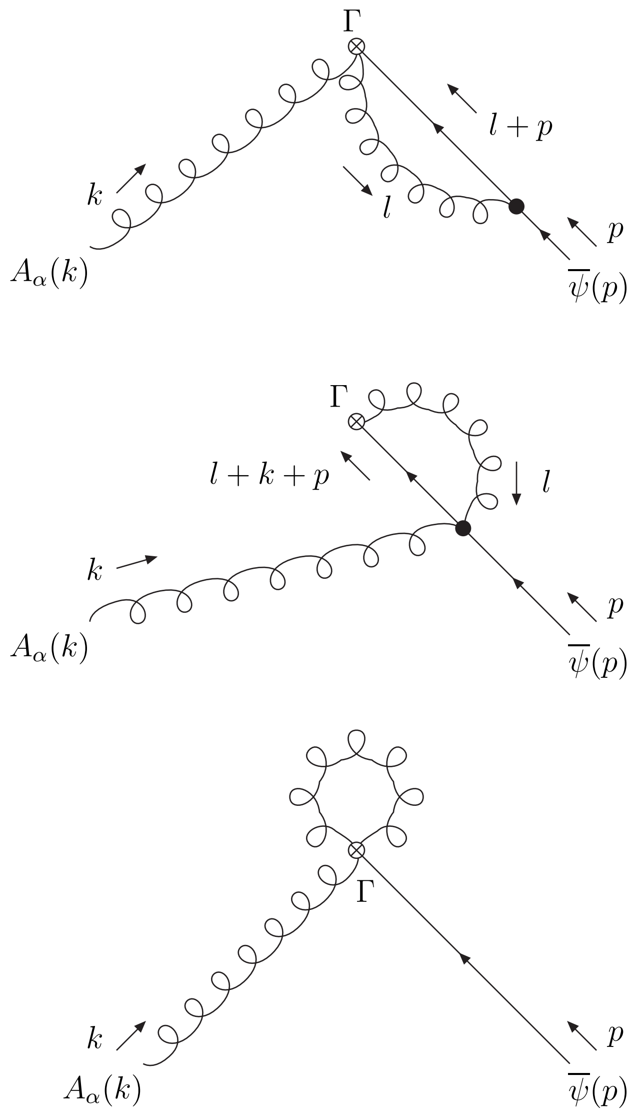
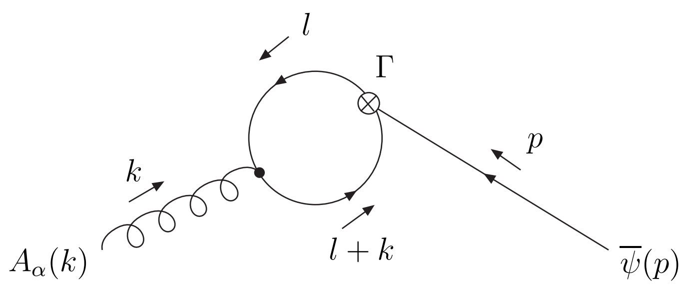


FIG. 3. One loop diagrams which contribute to the operator vertex correction in the SUSY WT identity. $\Gamma = S_\mu, D_S, X_S^{(1-3)}$.



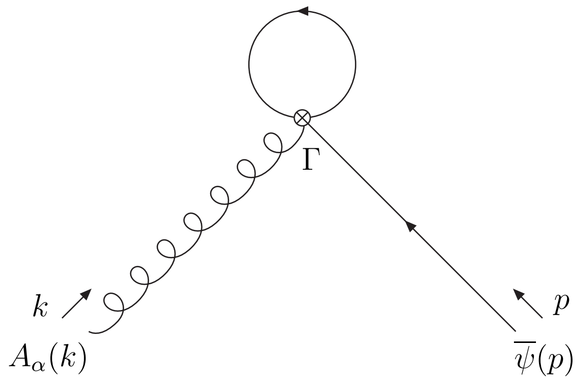


FIG. 4. One loop diagrams which contribute to the explicit SUSY breaking term; $\Gamma = X_S^{(4)}$.